Robertson-Walker Type Lyttleton-Bondi Universe in Five Dimensional General Theory of Relativity

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Abstract

We study the Robertson-Walker type model in the Lyttleton-Bondi universe in five dimensional general theory of relativity. Some exact and physical properties of solution are discussed.

1 Introduction

Lyttleton and Bondi [1] have developed a cosmological model assuming that there is a continuous creation matter due to a net imbalance of charge. To incorporate the idea of creation, the Maxwell field equations are modified as

$$F_{ij} = A_{i,j} - A_{j,i} \tag{1}$$

$$F^{ij}_{;j} = J^i - \lambda A^i \tag{2}$$

$$J^i_{;i} = q\,; \tag{3}$$

where λ is a constant , A_i and J_i denote five potential and current density five vector respectively, F_{ij} denote the anti-symmetric electromagnetic field tensor and q the rate of creation of a charge per unit proper volume. A semicolon denotes covariant differentiation.
The energy momentum tensor of the field is of the form

$$T^j_i = \left(F_{i\,k}F^{k\,j} + \frac{1}{4}F_{k\,l}F^{k\,l}\right) + \lambda\left(A_{i\,j} - \frac{1}{2}\,\delta^j_i A_k A^k\right) \tag{4}$$

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where $F_{ij} = 0$ (with zero electromagnetic field) relation

$$J_i = \lambda\, A_{i;} \tag{5}$$

and the energy momentum tensor of the field becomes

$$T_i^j = \lambda \left(A_i\, A^j - \frac{1}{2}\, \delta_i^j A_k A^k \right) \tag{6}$$

Assuming that the charge created will not affect the dynamical characteristic of the metric and the mechanical affect of such creation on the energy momentum tensor is nil, we can use the Einstein's field equations

$$R_i^j - \frac{1}{2}\, \delta_i^j R = -\kappa T_i^j \tag{7}$$

where $\kappa (= 8\pi G)$ is a constant. The velocity of light c is taken to be unity and T_i^j is given by equation (6).
Lyttleton and Bondi [1], Burman [2], Nduka [3], Rao and Panda [4], Reddy and Rao [5] are some of the authors who have investigated several aspects of the Lyttleton -Bondi field.

In this paper we consider the Kaluza-Klein type Robertson-Walker(Chaterjee[6]) model in the Lyttleton-Bondi universe. Some physical properties of the model are discussed. This result is an extension of a similar one obtained by Reddy [7].

1.1 Solutions of Field equations

$$ds^2 = dt^2 - R^2(t) \left[(1 + \frac{b}{r} - lr^2)^{-1} dr^2 + r^2 (d\theta_1^2 + sin^2\theta_1 d\theta_2^2) + (1 + \frac{b}{r} - lr^2) dy^2 \right] \tag{8}$$

where $l = +1, 0, -1$ for a closed, flat or open space, respectively. In this situation A_i must have the form given by

$$A_i = (a, 0, 0, 0, \phi) \tag{9}$$

But since $F_{ij} = 0$, we have $\frac{\partial a}{\partial t} = \frac{\partial \phi}{\partial r}$.
From equation from (8) and (9)

$$A^i = \left[-(\frac{1 + \frac{b}{r} - lr^2)}{R^2}) a,\ 0,\ 0,\ 0,\ \phi \right] \tag{10}$$

so that

$$A_i A^i = \left[-(\frac{1 + \frac{b}{r} - lr^2)}{R^2}) a^2 + \phi^2 \right] \tag{11}$$

By use of equations (9) and (11), the field equation (7) for the metric (8) can be written as

$$\frac{3}{R^2}(R\ddot{R} + \dot{R}^2 + l) = -\frac{\kappa\lambda}{2} \left[(\frac{1 + \frac{b}{r} - lr^2)}{R^2}) a^2 + \phi^2 \right] \tag{12}$$

$$\frac{3}{R^2}(R\ddot{R} + \dot{R}^2 + l) = \frac{\kappa\lambda}{2}\left[(\frac{1 + \frac{b}{r} - lr^2)}{R^2})a^2 - \phi^2\right] \tag{13}$$

$$\frac{6}{R^2}(\dot{R}^2 + l) = \frac{\kappa\lambda}{2}\left[(\frac{1 + \frac{b}{r} - lr^2)}{R^2})a^2 + \phi^2\right] \tag{14}$$

$$\lambda\ a\ \phi\left(\frac{1 + \frac{b}{r} - lr^2)}{R^2}\right) = 0 \tag{15}$$

where an overhead dot denotes difference with respect to the time t. Subtracting equation (12) from equation (13), we get $a = 0$ which in view of equation (10) implies $\phi = \phi(t)$. Now equation (12) to (15) reduces to

$$R\ddot{R} + \dot{R}^2 + l = -(\frac{\kappa\lambda}{6})R^2\phi^2 \tag{16}$$

$$\dot{R}^2 + l = \frac{\kappa\lambda}{12}R^2\phi^2 \tag{17}$$

The exact solution of the set of field equations (16) for $l = 0$ can be written as

$$R(t) = (At + B)^{\frac{1}{4}},\ \ \phi = \pm\sqrt{\frac{3A^2}{4\kappa}}(At + B)^{-1} \tag{18}$$

where A and B are constants of integration. The corresponding metric of the solutions can now be written as

$$ds^2 = dt^2 - (At + B)^{\frac{1}{2}}\left[(1 + \frac{b}{r})^{-1}dr^2 + r^2(d\theta_1^2 + sin^2\theta_1 d\theta_2^2) + (1 + \frac{b}{r})dy^2\right] \tag{19}$$

This model is, in general, non-static, homogeneous and isotropic. The following are some of the properties of the space time.

1. For $B > 0, A > 0$ the flat 4-space expands indefinitely from initial singular state.

2. For $B > 0, A < 0$ the initially finite flat universe contracts to a singular condition in a finite time.

3. It is interesting to note that in case (1) ϕ as given by equation (18) decreases with time where as in case (2) it increases indefinitely with time.

For $l = \pm 1$ no general solution to equations (1) could be obtained.

References

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